Practice test

reCalculus 12A Exponential Functions:

ame: _____ Key

off 1 only give 2 #5 only give 2

Part A Multiple Choice:

B 1. What is the value of $(3^0 + 4^{-1})^{-2}$?

A. 16/169 B. 16/25 C. 25/16 D. 4

$(3^0 + 4^{-1})^{-2}$ → $(1 + \frac{1}{4})^{-2}$ → $(\frac{4}{4} + \frac{1}{4})$ → $(\frac{5}{4})^{-2}$ → $(\frac{4}{5})^2$ → $\frac{16}{25}$

D 2. Sarah's father age is given by the expression $\frac{a^{x+4} \cdot a^{3x+1}}{a^{4x+3}}$, when a=6. How old is Sarah's father

A. 54 B. 42 C. 48 D. 36

$\frac{a^{x+4+3x+1}}{a^{4x+3}} \Rightarrow \frac{a^{4x+5}}{a^{4x+3}} \Rightarrow a^{4x-4x+5-3} \Rightarrow a^2 = 6^2 =$

D 3. A new computer is said to lose 30% of its value every 6 months. If the computer cost $2500 initially which equation show the value (V) of the computer after t years?

(1 − 0.3) = 0.7

A. $V = 2500(0.30)^{\frac{t}{2}}$ B. $V = 2500(0.70)^{\frac{t}{2}}$ C. $V = 2500(0.30)^{2t}$ D. $V = 2500(0.70)^{2t}$

2t is # of 6 month periods in t-years or $t/0.5 \Rightarrow 2t$

B 4 A material has an initial mass of 50 mg and a half-life of 20 minutes. Which expression represents the mass after t minutes?

$50(\frac{1}{2})^{\frac{t}{20}}$

A. $50 \times 2^{\frac{t}{20}}$ mg B. $50 \times 0.5^{\frac{t}{20}}$ mg C. 50×0.5^{20t} mg D. $20 \times 0.5^{\frac{t}{20}}$ mg

A 5. What are the coordinates of the x-intercept of the function $y = -2(3)^{2x} + 54$?

A. (1.5, 0) B. (54, 0) C. (2/3, 0) D. (3,0)

$0 = -2(3)^{2x} + 54$

$\frac{-54}{-2} = \frac{-2(3)^{2x}}{-2}$

$+27 = 3^{2x}$

$3^3 = 3^{2x}$

$\therefore 2x = 3$

$x = 3/2$

$x = 1.5$

C 6. Solve: $16^{2x-1} = 64^{x+3}$?

A. x=2 B. x=11 C. x= -11/4 D. x=4

$(2^4)^{2x-1} = (2^6)^{x+3}$

$2^{8x-4} = 2^{6x+18}$

$\therefore 8x - 4 = 6x + 18$

$8x - 6x = 18 + 4$

$2x = 22$

$x = 11$

C 7. Express as a single power of 3 $\frac{3^{n+1} \times 9^{n+1}}{27^n}$

A. 3^{2n+6} B. 3^{16} C. 3^3 D. 3^{n+3}

$\frac{3^{n+1} \times 3^{2(n+1)}}{3^{3n}} = \frac{3^{n+1+2n+2}}{3^{3n}} = \frac{3^{3n+3}}{3^{3n}} = 3^{3n-3n+3} = 3^3 = 27$

B 8. Jason has been collecting die-cast cars for several years and realizes that the value of each car increases exponentially with time. One car in his collection was purchased for $1.80 in 1970 and its value has doubled every 6 years since then. Which of the following equations could be used to model the value of the car (V) in terms of the number of years (t) since it was purchased?

A. $V = 1.80(2)^{6t}$ ← 6 times a year B. $V = 1.80(2)^{\frac{t}{6}}$ every 6 yrs C. $V = 2(1.80)^{6t}$ D. $V = 2(1.80)^{\frac{t}{6}}$

A 9. Simplify $\frac{\left(a^{\frac{1}{4}}\right)^3\left(a^{\frac{5}{6}}\right)}{\left(a^{\frac{2}{3}}\right)^4\left(a^{\frac{11}{12}}\right)}$

A. a^{-2} B. $a^{-\frac{127}{72}}$ C. $a^{-\frac{5}{24}}$ D. a^5

$a^{\left(\frac{3}{4} + \frac{5}{6}\right) - \left(\frac{8}{3} + \frac{11}{12}\right)}$

$a^{\left(\frac{9}{12} + \frac{10}{12}\right) = \left(\frac{32}{12} + \frac{11}{12}\right)}$

$a^{\left(\frac{19}{12}\right) - \left(\frac{43}{12}\right)}$

$a^{\left(\frac{-24}{12}\right)}$

a^{-2}

a^{-2} so

10. Simplify $\frac{(2x^2y^2)^5}{2(8x^4y^3)(x^2y)^3}$

A. x^4 B. $\frac{5}{8}x^4$ C. $2xy$ D. $2y^4$

$\frac{2^5 x^{10} y^{10}}{2(2^3)x^4y^3x^6y^3} = \frac{2^5 x^{10} y^6}{2^4 x^{10} y^6} = 2^1 y^4$

Part B

1. Evaluate

a. $2^{-1}(3^3-2^0)$

$2^{-1}(27-1) = \frac{1}{2}(26) = \boxed{13}$

b. $(125^{2/3}+16^{3/2}-64^{1/2})^{1/2}$

$\sqrt{((\sqrt[3]{125})^2+(\sqrt{16})^3-(\sqrt{64}))}$

$\sqrt{((5)^2+(4)^3-8)}$

$\sqrt{(25+64-8)}$

$\sqrt{81} = \boxed{9}$

c. $36^{\frac{1}{2}}+\sqrt[3]{64}-5^0$

$6+4-1 = \boxed{9}$

d. $\frac{9^{\frac{2}{3}}\cdot 9^{3/3}}{9^{\frac{1}{6}}} = \frac{9^{5/3}}{9^{1/6}} = \frac{9^{10/6}}{9^{1/6}} = 9^{10/6-1/6} = 9^{9/6} = 9^{3/2} = (\sqrt{9})^3 = (3)^3 = \boxed{27}$

3. Solve

a. $27^{2x} = 81$

$(3^3)^{2x} = (3^4)$

$3^{6x} = 3^4$

$\therefore 6x = 4$

$x = 4/6$

$\boxed{x = 2/3}$

b. $2^{3a+2} = \frac{1}{16^{2a+1}}$

$2^{3a+2} = 2^{-4(2a+1)}$

$= 2^{-8a-4}$

$\therefore 3a+2 = -8a-4$

$3a+8a = -4-2$

$11a = -6$

$a = -6/11$

c. $\sqrt[3]{32} = 2^{x-3}$

$(32)^{\frac{1}{3}}$

$(2^5)^{1/3} = 2^{x-3}$

$2^{5/3} = 2^{x-3}$

$\frac{5}{3} = x-3$

$\frac{5}{3}+\frac{3}{1} = x$

$\frac{5}{3}+\frac{9}{3} = x$

$\boxed{\frac{14}{3} = x}$

d. $125^{x^2+3x} = 25^{x+3}$

$(5^3)^{x^2+3x} = 5^{2(x+3)}$

$\therefore 3x^2+9x = 2x+6$

$3x^2-2x+9x-6 = 0$

$3x^2+7x-6 = 0$

$3x^2+9x-2x-6 = 0$

$3x(x+3)-2(x+3) = 0$

$(3x-2)(x+3) = 0$

$x-3=0$ And $3x-2=0$

$\boxed{x=3}$ $3x=2$

$\boxed{x=2/3}$

4. a. Ralph buys a new house valued at $125 000. If the value of the house increases by 4.9% every 3 years, How much will the house be worth in 20 years?

b. Taylor just purchased stocks for $1250, the value of his stock has been increasing at a rate of 6% three times a year. What will his stock be worth in 5 years?

c. A bacteria culture starts with 25 cells and doubles every 4 hours, how many will there be in 5 days?

d. 500 mg A medication is given once a week to treat a certain viral infection. If the medication decreases by 16% every 5 hours, how much remains in the body after one week.

↳ 7 weeks =

5. Graph the following. State the intercepts, HA, Domain and range

a. $y = 2(3)^{x+2}-18$

b. $y = -3(4)^{x-1}+12$

c. $y = -0.5(6)^{x+2}+108$

d. $y = 3(2)^{2x+1}-96$

→ on paper

4a) $V(t) = 125\,000\,(1.049)^{t/3}$

$= 125000\,(1.049)^{20/3}$

$= 171\,954.39$

b) $S(t) = 1250\,(1.06)^{3t}$

$= 1250\,(1.06)^{3(5)}$

$= 1250\,(1.06)^{15}$

$= \$2995.70$

c) $B(t) = 25\,(2)^{4t}$

$= 25\,(2)^{4(120)}$

$= 25\,(2)^{480}$

$= 7.8 \times 10^{145}$ bacteria

5 days is 5×24 hrs $= 120$ hrs

d) $m(t) = 500\,(0.84)^{\frac{t}{5}}$

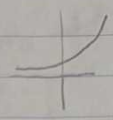
$= 500\,(0.84)^{\frac{168}{5}}$

$= 500\,(0.84)^{33.6}$

$= 1.43$ mg is left

$t = 1$ week $= 7$ days $\times 24$ hrs $= 16$

Part B

5a) $y = 2(3)^{x+2} - 18$ Growth 

x-int

$0 = 2(3)^{x+2} - 18$

$18 = 2(3)^{x+2}$

$\frac{18}{2} = \frac{2}{2}(3)^{x+2}$

$9 = 3^{x+2}$

$3^2 = 3^{x+2}$

$\therefore$

$2 = x+2$

$x = 2-2$

$x = 0$

$(0,0)$

y intercept

$y = 2(3)^{0+2} - 18$

$= 2(3)^2 - 18$

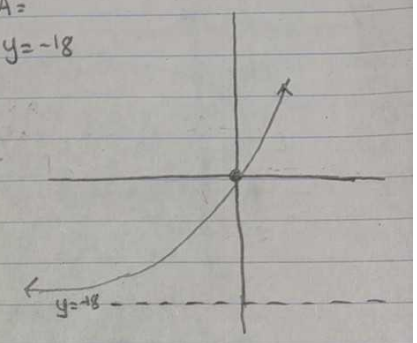
$= 2(9) - 18$

$= 18 - 18$

$= 0$

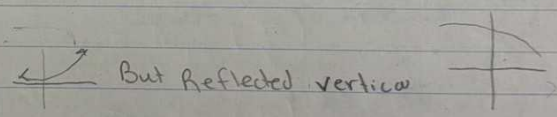
$(0,0)$

HA: $y = -18$



Domain $x \in R$

Range $y > -18$, $y \in R$

b) $y = -3(4)^{x-1} + 12$ ↑ growth  But Reflected vertical

$y = 4^x$

x	y
-2	0.0625
-1	0.25
0	1
1	4
2	16

↓ mapping

$(x,y) = (x+1, -3y+12)$

x	y
-1	11.8125
0	11.25
1	
2	
3	

x int

$0 = -3(4)^{x-1} + 12$

$-12 = -3(4)^{x-1}$

$\frac{-12}{-3} = \frac{-3}{-3}(4)^{x-1}$

$4 = 4^{x-1}$

$\therefore$

$1 = x-1$

$1+1 = x$

$x = 2$

$(2,0)$

y int

$y = -3(4)^{0-1} + 12$

$y = -3(4)^{-1} + 12$

$= -3(\frac{1}{4}) + 12$

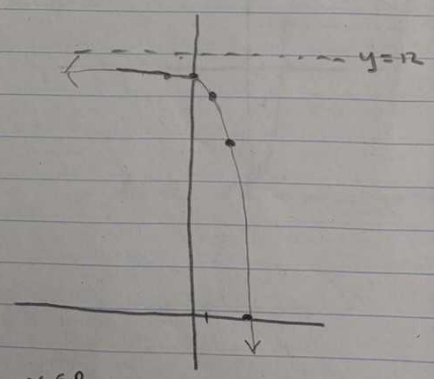
$= \frac{-3}{4} + \frac{12}{1}$

$= \frac{-3}{4} + \frac{48}{4}$

$= \frac{47}{4}$

$= 11.75$

HA $\Rightarrow y = 12$



D $x \in R$

R: $y < 12$, $y \in R$

5c) $y = -0.5(6)^{x+2} + 108$

Growth ≠ But Reflect vertically

x intercept

$0 = -0.5(6)^{x+2} + 108$

$-108 = -0.5(6)^{x+2}$

$\frac{-108}{-0.5} = \frac{-0.5}{-0.5}(6)^{x+2}$

$216 = 6^{x+2}$

$6^3 = 6^{x+2}$

$\therefore$

$3 = x+2$

$3-2 = x$

$\boxed{x = 1}$

$(1,0)$

y intercept

$y = -0.5(6)^{0+2} + 108$

$y = -0.5(6)^2 + 108$

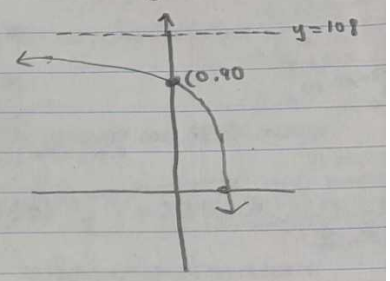
$y = -0.5(36) + 108$

$y = -18 + 108$

$y = 90$

$(0, 90)$

HA => $y = 108$



D: $x \in R$

R: $y < 108$ $y \in R$

5d) $y = 3(2)^{2x+1} - 96$

Growth No Reflection

x int

$0 = 3(2)^{2x+1} - 96$

$96 = 3(2)^{2x+1}$

$\frac{96}{3} = \frac{3(2)^{2x+1}}{3}$

$32 = 2^{2x+1}$

$2^5 = 2^{2x+1}$

$\therefore$

$5 = 2x+1$

$5-1 = 2x$

$4 = 2x$

$\frac{4}{2} = \frac{2x}{2}$

$\boxed{x = 2}$

$(2,0)$

y int

$y = 3(2)^{2(0)+1} - 96$

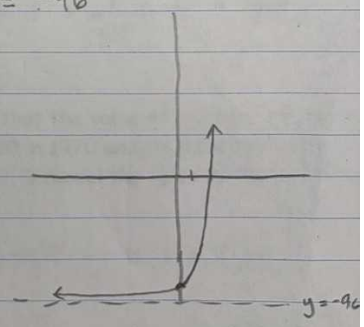
$= 3(2)^1 - 96$

$= 6 - 96$

$= -90$

$(0, -90)$

HA = -96



Domain $x \in R$

Range $y > -96$, $y \in R$