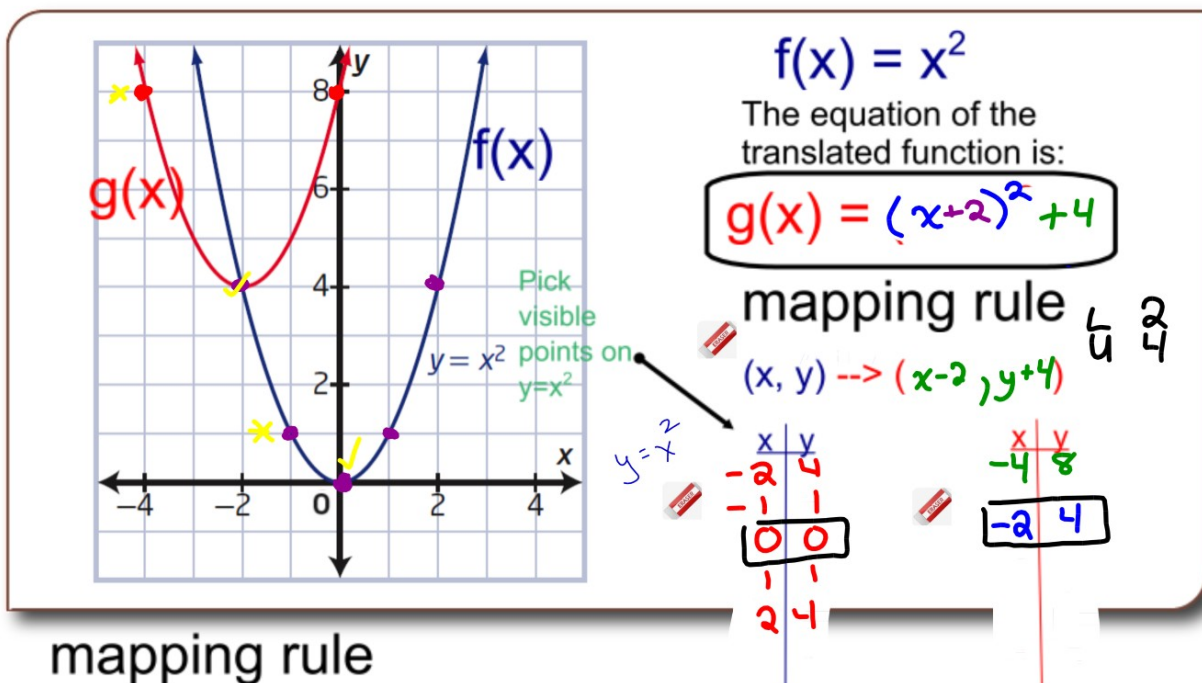


Chapter

1

Horizontal and Vertical Translations



HW solutions

horizontal translation HT Left 7	vertical translation VT Down 3	HT Right 5 VT Up 9
$y=f(x)$ $y=f(x+7)$	$y=f(x)-3$	$y=f(x-5)+9$
$y=x^2$ $y=(x+7)^2$	$y=x^2-3$	$y=(x-5)^2+9$
$y=x^3$ $y=(x+7)^3$	$y=x^3-3$	$y=(x-5)^3+9$
$y= x $ $y= x+7 $	$y= x -3$	$y= x-5 +9$
$y=\sqrt{x}$ $y=\sqrt{x+7}$	$y=\sqrt{x}-3$	$y=\sqrt{x-5}+9$
$y=\frac{1}{x}$ $y=\frac{1}{x+7}$	$y=\frac{1}{x}-3$	$y=\frac{1}{x-5}+9$
$y=3^x$ $y=3^{x+7}$	$y=3^x-3$	$y=3^{x-5}+9$
$y=\sin x$ $y=\sin(x+7)$	$y=\sin x-3$	$y=\sin(x-5)+9$

Do we need more? HW solutions K h

	 Vertical Translation	 Horizontal Translation
$f(x) = \cos(x - 3) + 6$	Up 6	Right 3
$f(x) = (x - 7)^3 - 9$	Down 9	Right 7
$y = 9^{x+8} - 12$	Down 12	Left 8
$y = \sqrt{x + 10} + 4$	Up 4	Left 10
$f(x) = \cos(x - 3) + 6$	Up 6	Right 3
$f(x) = (x + 5.7)^2 - 2$	Down 2	Left 5.7
$y = x - 15$	Down 15	—
$y = \tan(x + 9)$	—	Left 9
$y = f(x - 8) + 4$	Up 4	Right 8

Copy this down



Function	Unit Function	Translation	Mapping Rule
$y=(x-6)^2+3$	$y=x^2$	Right 6 Up 3 HT=6 VT=3	$(x,y) \rightarrow (x+6, y+3)$

This means that a unit quadratic will be translated right 6 and up 3

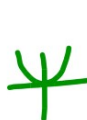
$$y=(x-6)^2+3$$

Write the mapping rule

Original Translated
 $(x,y) \rightarrow (x+6, y+3)$

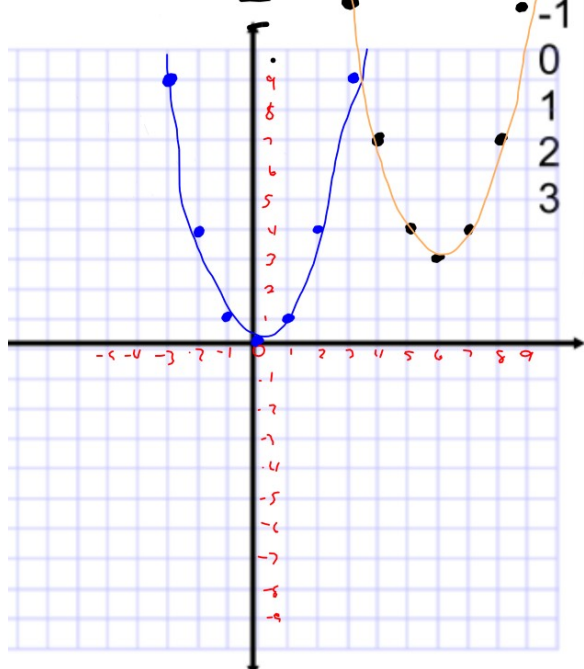
What are the translations?

Write ordered pairs from $x = -3$ to $+3$.
Then the mapping points



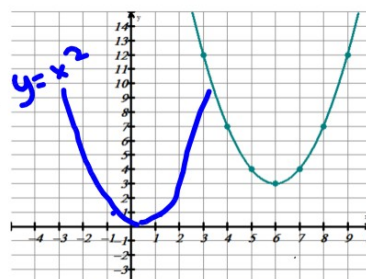
$$y=x^2$$

What is the unit function?



x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

x	y
3	12
4	7
5	4
6	3
7	4
8	7
9	12



Move blue see neat drawing

Use graph paper to graph the mapped points

Skip if they are getting it

RECALL: A mapping rule can be used to map a set of ordered pairs from a function onto a translated function

For Horizontal and Vertical translations

$$y=f(x-h)+k$$

horizontal
translation

vertical
translation



to see answers



Function

unit
function

Translation

Mapping Rule

$$y=(x+5)^2-11$$

$$y=$$

$$y=x^3+18$$

$$y=$$

$$y=f(x+6)$$

$$y=$$

$$y=(x-12.6)^3-8$$

$$y=|x-10|+7$$

$$y=\sqrt{x+5}+6$$



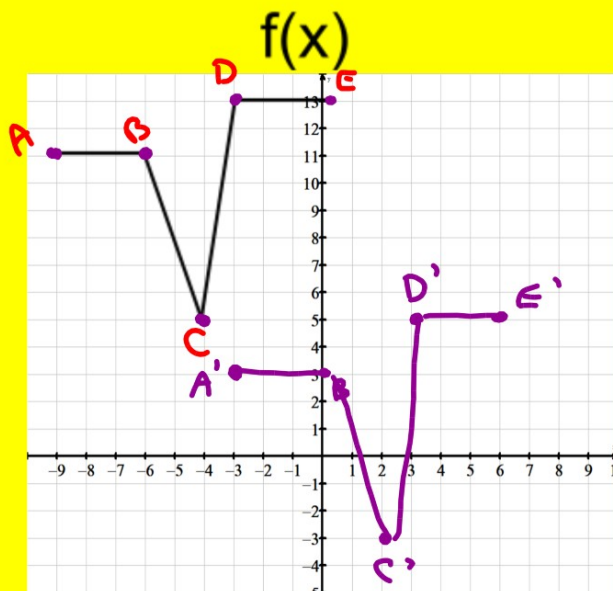
Sketch $g(x)=f(x-6)-8$

Answer on next slide

VT $\mathcal{D}8$
HT $R6$

$$g(x)=f(x-6)-8$$

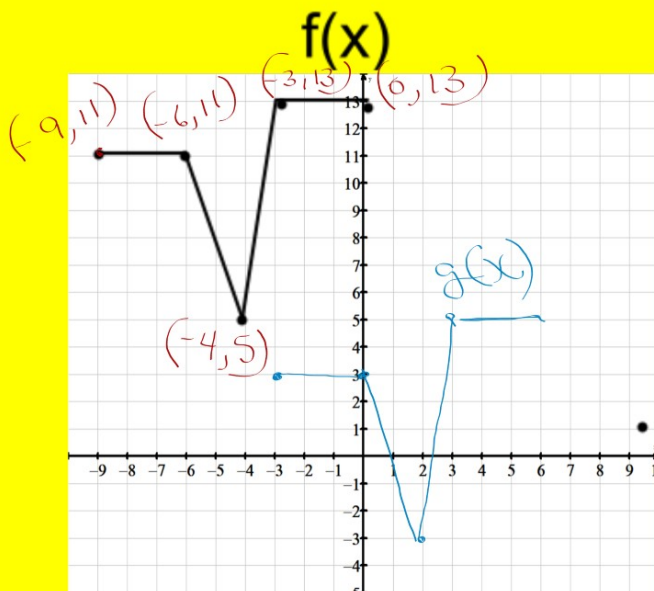
$$(x,y) \rightarrow (x+6, y-8)$$



	x	y
A	-9	11
B	-6	11
C	-4	5
D	-3	13
E	0	13

	x	y
A'	-3	3
B'	0	3
C'	2	-3
D'	3	5
E'	6	5

Sketch $g(x)=f(x-6)-8$



$$g(x)=f(x-6)-8$$

$$(x, y) \rightarrow (x+6, y-8)$$

R6
D8

	x	y
A	-9	11
B	-6	11
C	-4	5
D	-3	13
E	0	13

	x	y
A'	-3	3
B'	0	3
C'	2	-3
D'	3	5
E'	6	5

Graph the original then the translates. 

$y = |x|$ translated $y = |x+3|+5$

Answer on
next slide

a) i) Make a table of vales for $y = |x|$

ii) Then table of values for $y = |x+3| + 5$

i) $y = |x|$

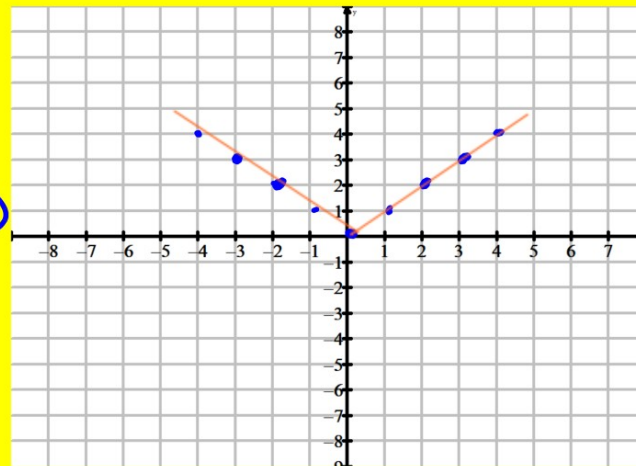
Basic

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

ii)

$(x, y) \rightarrow (x-3, y+5)$

x	y
-6	8
-5	7
-4	6
-3	5
-2	6
-1	7
0	8



Graph the original then the translates.

$$y = |x|$$

translated

$$y = |x+3|+5$$

SOLUTION

HT -3 3 left VT $+5$ up 5

a) i) Make a table of values for $y = |x|$

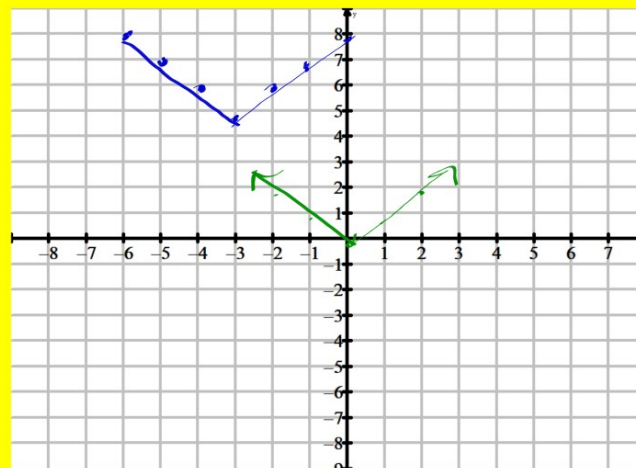
ii) Then table of values for $y = |x+3|+5$

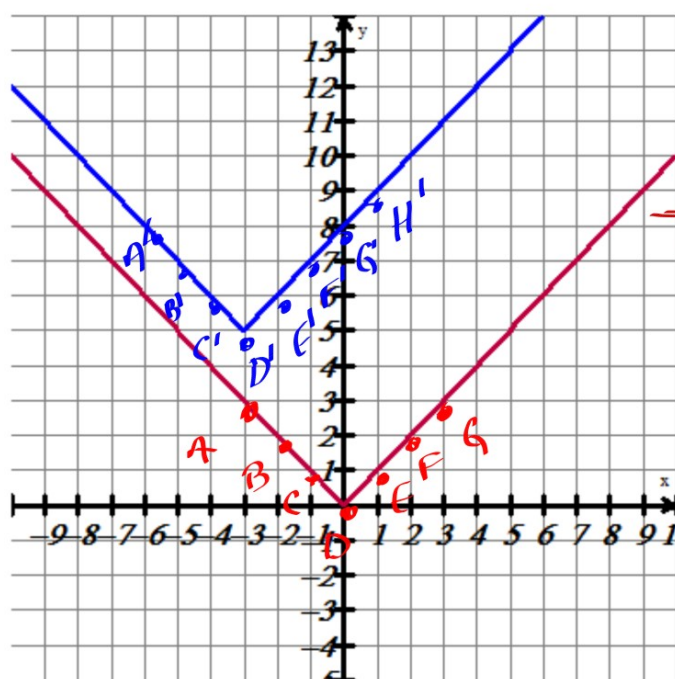
Graph

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

Graph: $(x, y) \rightarrow (x-3, y+5)$

x	y
-6	8
-5	7
-4	6
-3	5
-2	6
-1	7
0	8





Solution

$$\rightarrow y = |x|$$

$$y = |x + 3| + 5$$

$$(x, y) \rightarrow (x - 3, y + 5)$$

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	0

x	y
-6	8
-5	7
-4	6
-3	5
-2	6
-1	7

Given the equations, identify the unit equation, the translation and the mapping rule.

$$y = (x+11)^2 - 6$$

Translation

Unit function $y = x^2$

Mapping rule

$$(x, y) \rightarrow (x-11, y-6)$$

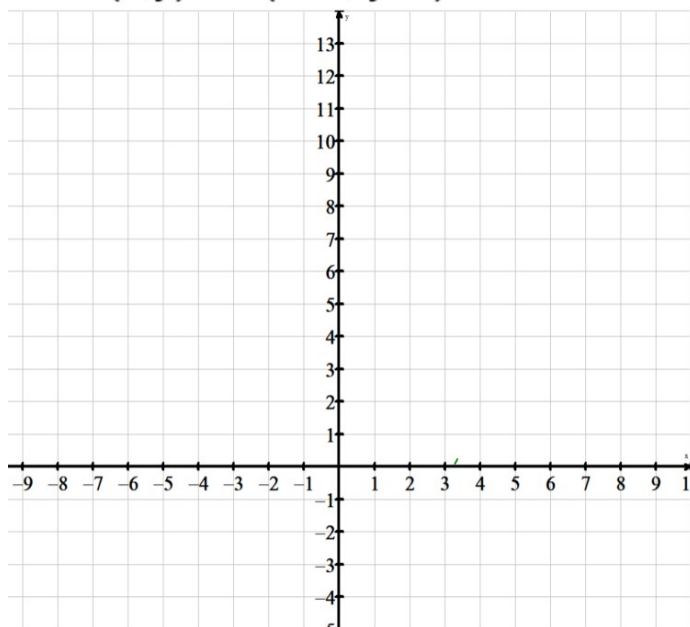
unit function

Equation	Translation	Mapping
$y = (x-4)^2 + 2$	R 4 U 2	$(x+4, y+2)$
$y = (x+2.4)^2 + 7$	L 2.4 U 7	$(x, y) \rightarrow (x-2.4, y+7)$
$y = (x-8)^2$	R 8 VT = None	$(x, y) \rightarrow (x+8, y)$
$y = f(x+3) + 8$		$(x, y) \rightarrow ($



$$y = f(x-h) + k$$

$$(x,y) \rightarrow (x+h, y+k)$$



Sketch $y = |x-5| + 4$

[Solution on next slide](#)

Start with $y = |x|$

$$(x,y) \rightarrow (\quad , \quad)$$

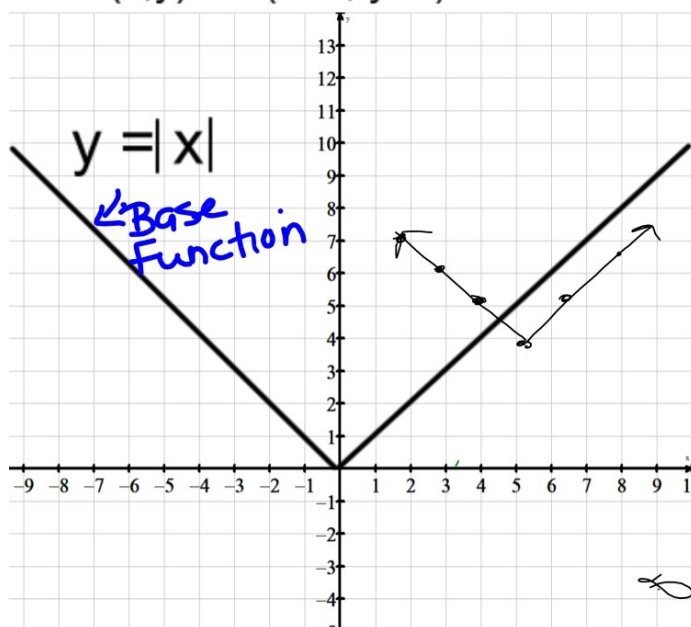
x	y
-1	
-2	
-3	
0	
1	
2	

x	y

$$y = f(x-h) + k$$

$$(x,y) \rightarrow (x+h, y+k)$$

Solution
Sketch



$$y = |x-5| + 4$$

Start with $y = |x|$

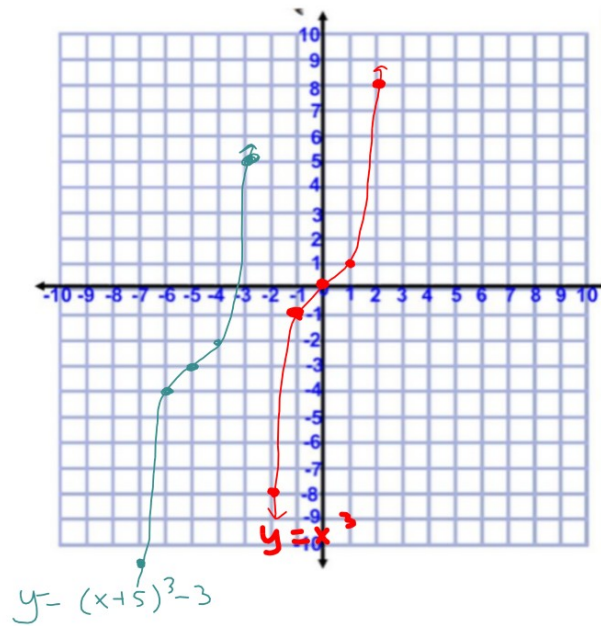
$$(x,y) \rightarrow (x+5, y+4)$$

x	y
-1	1
-2	2
-3	3
0	0
1	1
2	2

x	y
4	5
3	6
2	7
5	0
6	1
7	2

$$(2,2) \rightarrow (7,6)$$

OLD
PT



solutions on next page

Sketch $y = (x+5)^3 - 3$

* This is a translation
of $y = x^3$ (unit cubic function)

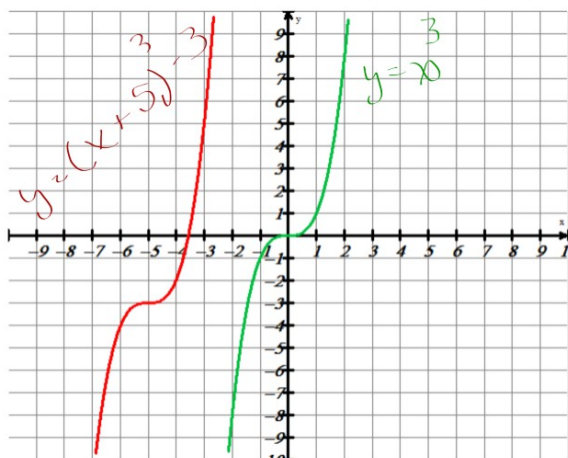
$$(x, y) \rightarrow (x-5, y-3)$$

$$y = x^3$$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

x	y
-7	-11
-6	-4
-5	-3
-4	-2
-3	5

Solution



Sketch $y = (x+5)^3 - 3$

* This is a translation

of $y = x^3$ (unit cubic function)

$$(x, y) \rightarrow (x-5, y-3)$$

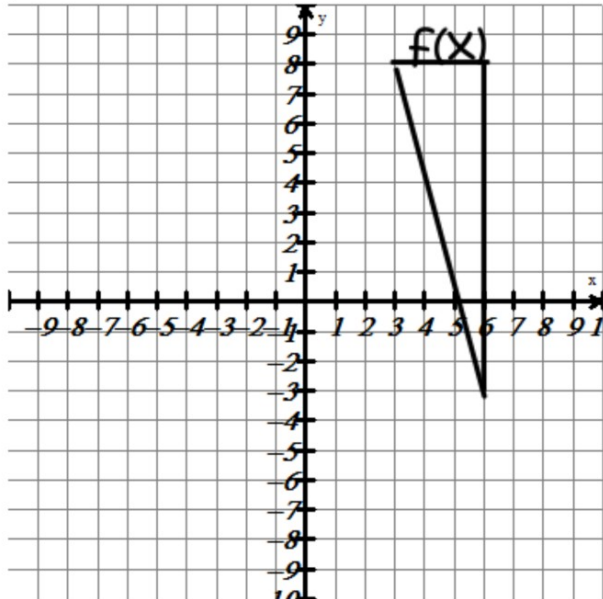
$$y = x^3$$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

x	y
-7	-11
-6	-4
-5	-3
-4	-2
-3	5



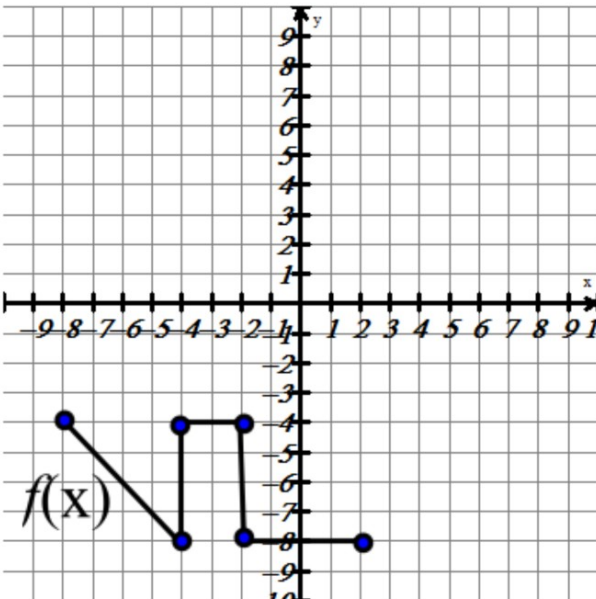
[solution on next slide](#)



$$h(x) = f(x+5)+2$$

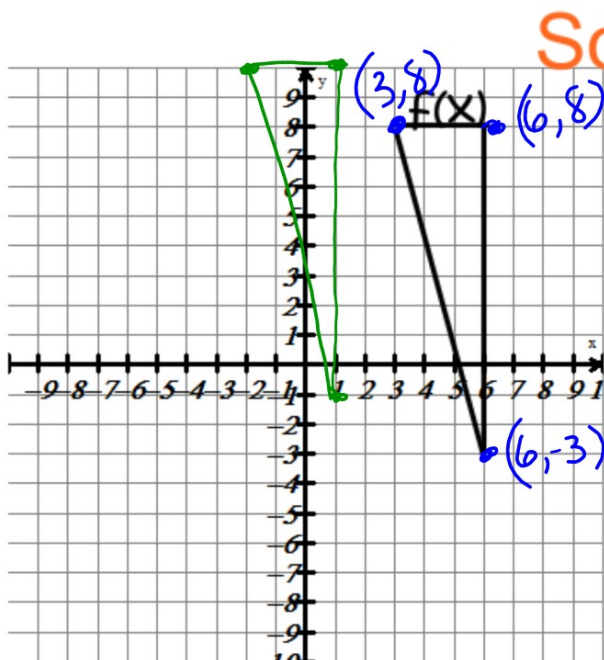
List the points of the new translated image $h(x)$

Draw the translated image $h(x)$



Graph

$$y = f(x-7)+10$$



Solution

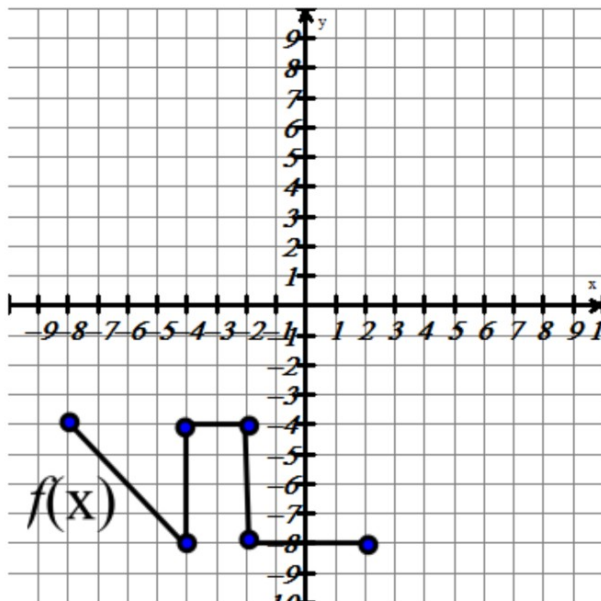
$$h(x) = f(x+5)+2$$

$$(x, y) \rightarrow (x-5, y+2)$$

x	y
3	8
6	8
6	-3

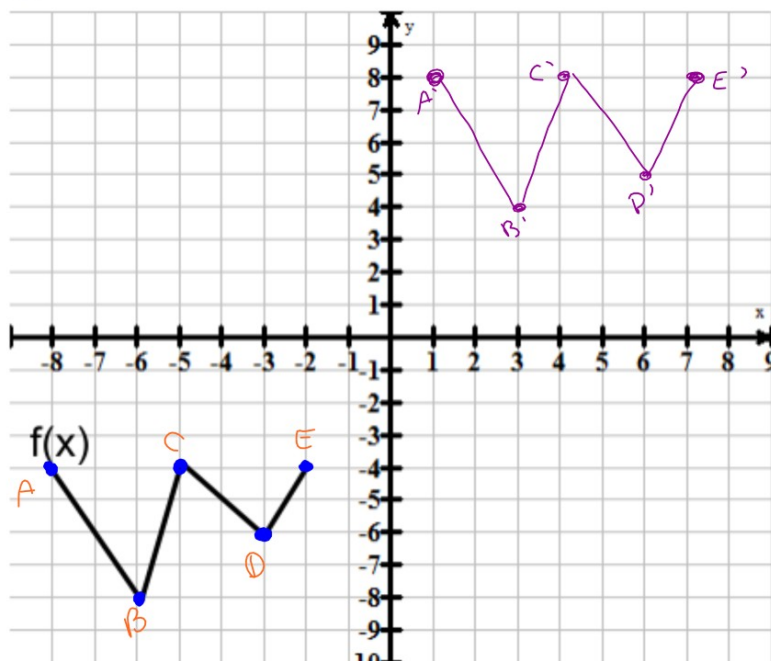
x	y
-2	10
1	10
1	-1

Left 5, up 2



Graph

$$y = f(x-7)+10$$



Given the graph $y = f(x)$
sketch the graph

$$y = f(x-9) + 12$$

Mapping $(x, y) \rightarrow (x+9, y+12)$

Original

	x	y
A	-8	-4
B	-6	-8
C	-5	-4
D	-3	-6
E	-2	-4

New

	x	y
A'	1	8
B'	3	4
C'	4	8
D'	6	6
E'	7	8

Original
Domain

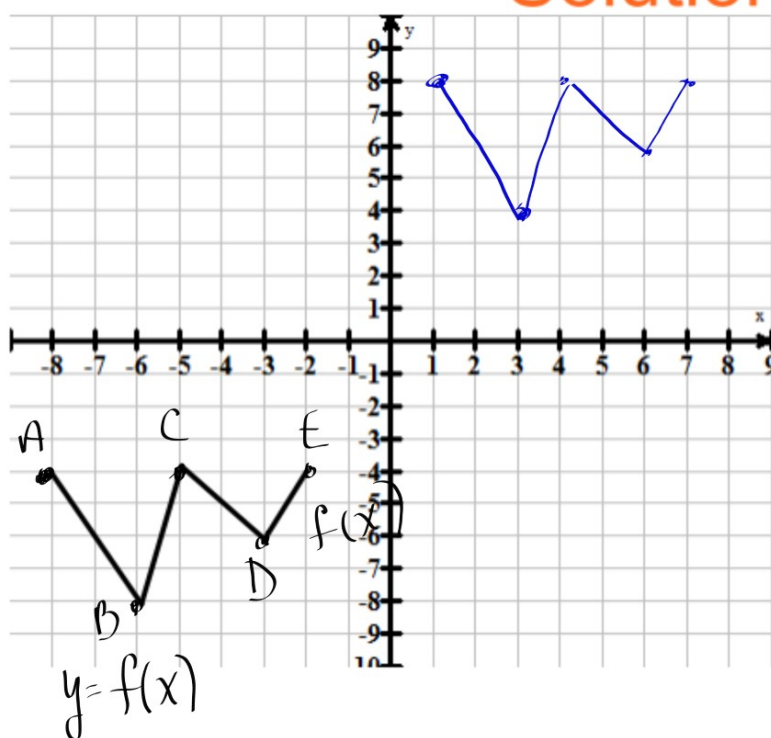
$\{ -8 \leq x \leq -2, x \in \mathbb{R} \}$

Range $\{ -8 \leq y \leq -4, y \in \mathbb{R} \}$

Transformed $y = f(x-9) + 12$
Domain $\{ 1 \leq x \leq 7, x \in \mathbb{R} \}$

Range $\{ 4 \leq y \leq 8, y \in \mathbb{R} \}$

Solution



Given the graph $y = f(x)$
sketch the graph

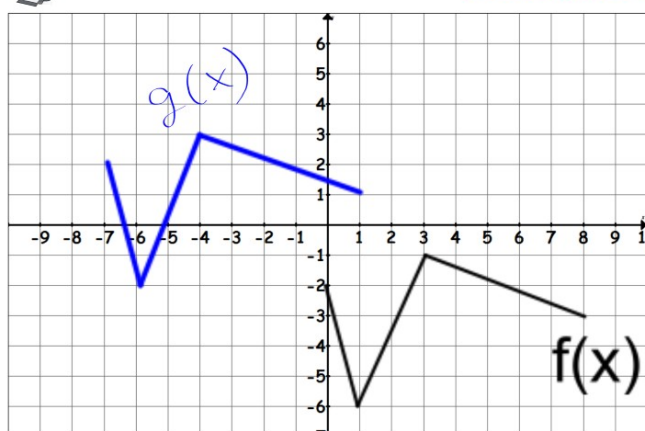
$$y = f(x-9) + 12$$

$$(x, y) \rightarrow (x+9, y+12)$$

A		A'	
x	y	x	y
-8	-4	1	8
-6	-8	3	4
-5	-3	4	8
-3	-6	6	6
-2	-4	7	8



Domain & Range



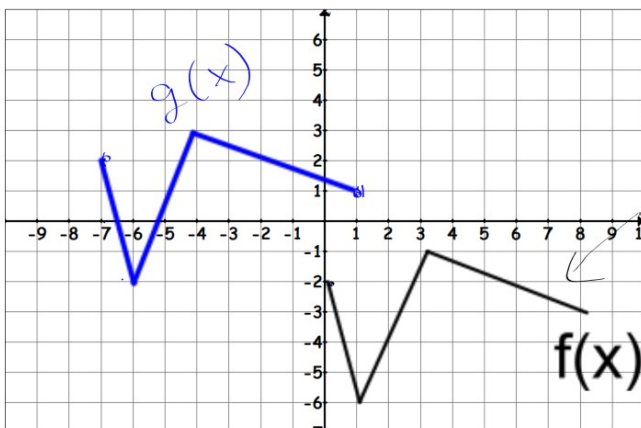
Sketch:

$$g(x) = f(x+7) + 4$$

$$(x, y) \longrightarrow (x-7, y+4)$$

x	y	x	y
0	-2	-7	2
1	-6	-6	-2
3	-1	-4	3
8	-3	1	1

now lets look at the **Domain** and **Range**:
 allowable values of x allowable values of y

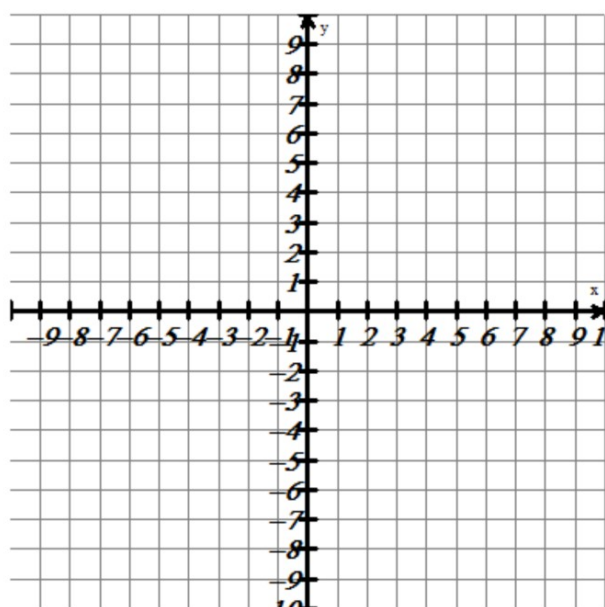
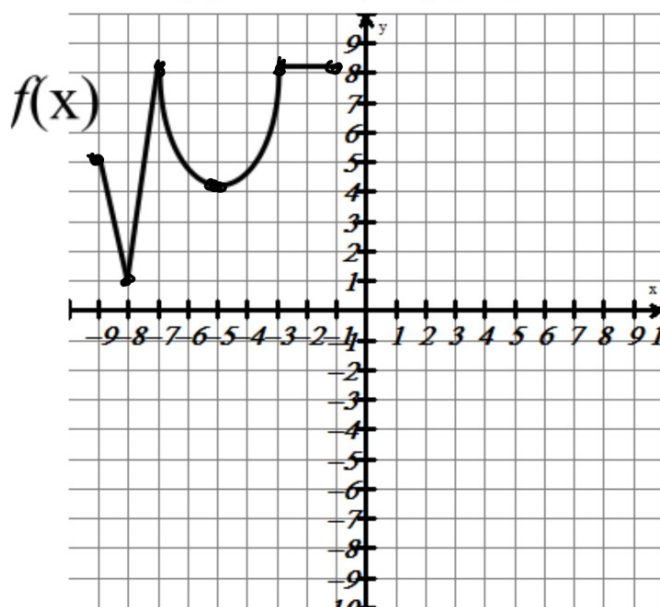


$f(x)$ Domain: $[-1, 8]$
 Range: $[-6, 3]$

$g(x)$ Domain: $[-7, 1]$
 Range: $[-2, 3]$

Only do next slides if needed.
(If not Jump to 57 which is Homework)

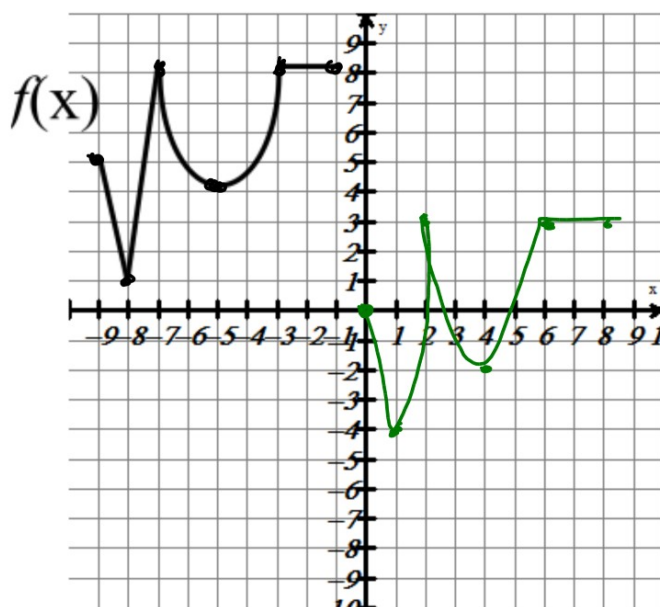
Given $f(x)$ sketch $y = f(x-9)-5$



Graph a basic quadratic function $y = x^2$

On the same graph sketch $y = (x+4)^2 - 6$

Given $f(x)$ sketch $y = f(x-9)-5$



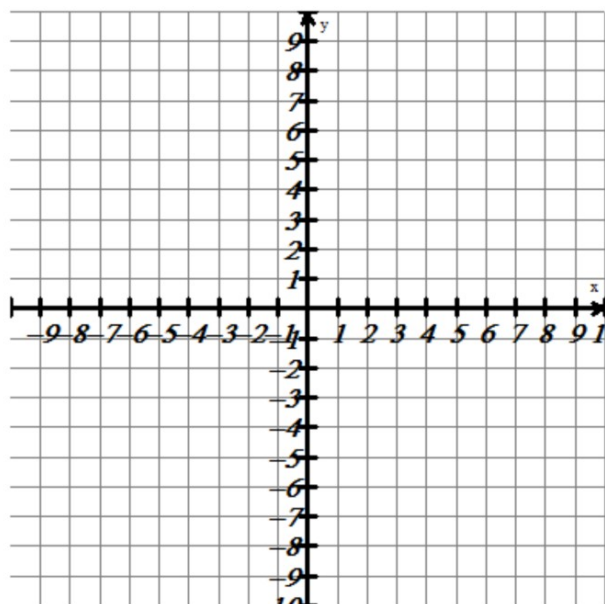
Solution

$$y = f(x-9)-5$$

$$(x, y) \rightarrow (x+9, y-5)$$

x	y
-9	5
-8	1
-7	8
-5	4
-3	8
-1	8

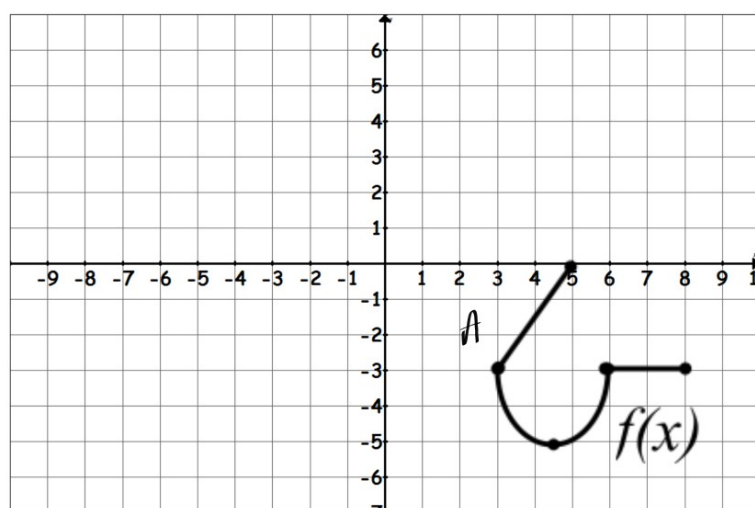
x	y
0	0
1	-4
2	3
4	-1
6	3
8	3



Graph a basic quadratic function $y = x^2$

On the same graph sketch $y = (x+4)^2 - 6$

Given $f(x)$ sketch $y = f(x+10)+6$. And state the domain and range of A) $f(x)$ and b) then of the new graph.



$$y = f(x+10) + 6$$

A) Domain:

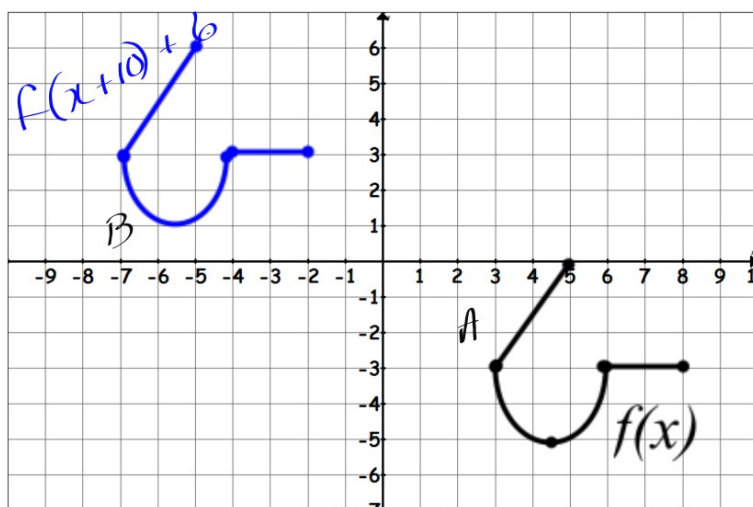
Range:

B) Domain:

Range:

Given $f(x)$ sketch $y = f(x+10)+6$. And state the domain and range of A) $f(x)$ and b) then of the new graph.

Solutions



$$y = f(x+10) + 6$$

$$(x, y) \rightarrow (x-10, y+6)$$

x	y
5	6
3	-3
4.5	-5
6	-3
8	-3

x	y
-5	6
-7	3
-4.5	1
-4	3
-2	3

A: Domain: $3 \leq x \leq 8$
Range: $-5 \leq y \leq 0$

B: Domain: $-7 \leq x \leq -2$
Range: $1 \leq y \leq 6$

1) Given $f(x) = \frac{1}{x}$
 translate 9 units right and 7
 units down, write the equation
 of the translated function

2) Given $f(x) = x^3$
 translate 15 units left, write the
 equation of the translated
 function

3) The function $y = x^2$ is
 translated 6 units left and
 4 units up, what is the
 equation of the translated
 function and the domain
 & range.

Do a quick sketch to figure
 out domain & range.

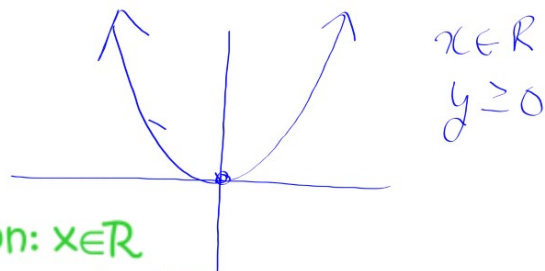
In PreCalc 11 you learned :

Domain of a quadratic function: $x \in \mathbb{R}$

Range of Quadratic function: $y \geq \min$ OR $y \leq \max$

4) What is the image of the point (0,0)
 under the translation $y = (x-7)^2 + 9$?

(Hint: Mapping)



Solution

- 1) Given $f(x) = \frac{1}{x}$
translate 9 units right and 7
units down, write the equation
of the translated function

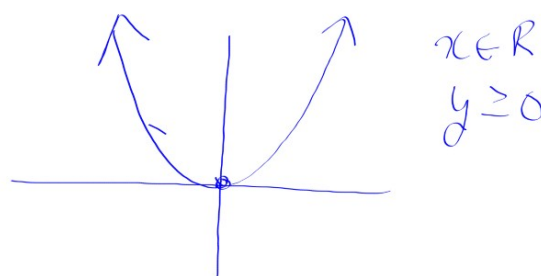
$$y = \frac{1}{x-9} - 7$$

- 2) Given $f(x) = x^3$
translate 15 units left, write the
equation of the translated
function

$$y = (x+15)^3$$

- 3) The function $y = x^2$ is
translated 6 units left and
4 units up, what is the
equation of the translated
function

$$y = (x+6)^2 + 4 \quad * \quad \begin{matrix} x \in \mathbb{R} \\ y \geq 4 \end{matrix}$$



In PreCalc 11 you learned :

Domain of a quadratic function: $x \in \mathbb{R}$

Range of Quadratic function: $y \geq \min$ OR $y \leq \max$

4a) What is the image of the point (0,0) under the
translation $y = (x-7)^2 + 9$?

(0,0) is on the original graph so do mapping.

$(x, y) \rightarrow (x+7, y+9)$

$(0,0) \rightarrow (7,9)$

Domain $\Rightarrow$ allowable values of x
 any polynomial OR absolute value
 function: $x \in \mathbb{R}$ [x can be any
 Real number]

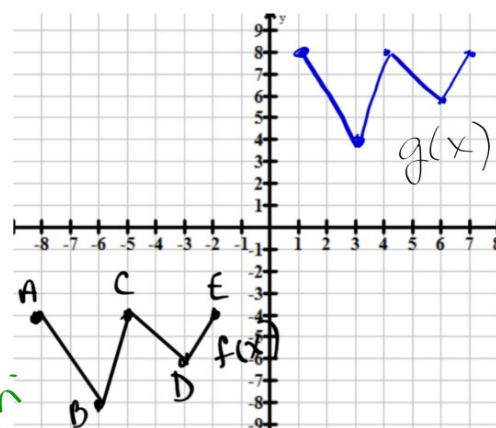
Range $\Rightarrow$ allowable values of y

$$g(x) = f(x-9) + 12$$

$f(x)$
 Domain: $-8 \leq x \leq -2$: $[-8, -2]$
 Range: $-8 \leq y \leq -4$: $[-8, -4]$

Interval notation

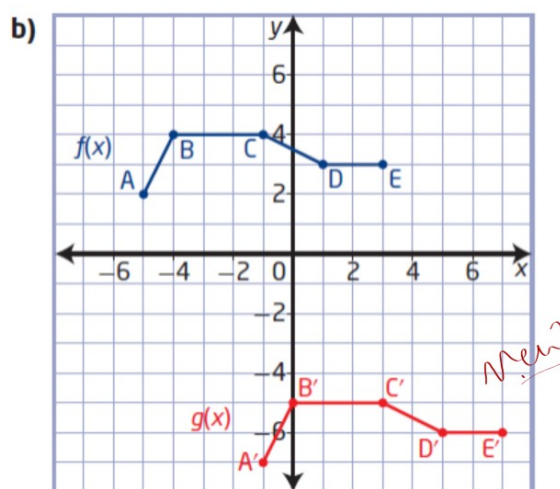
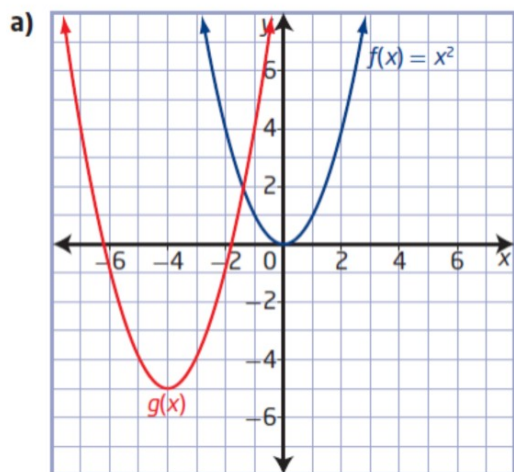
$$(x, y) \rightarrow (x+9, y+12)$$



$y = f(x)$
 $g(x)$: Domain $[1, 7]$
 Range $[4, 8]$

What transforms $f(x)$ to $g(x)$

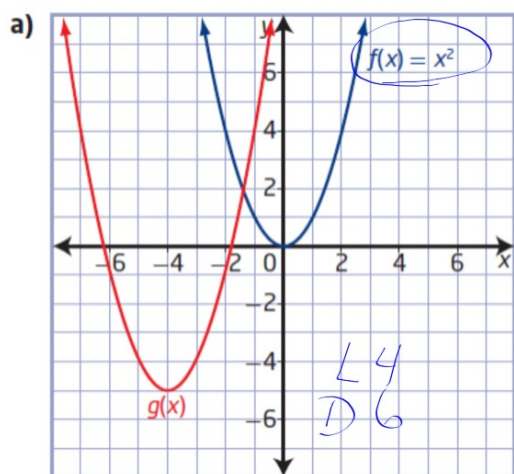
Determine the Equation of a Translated Function



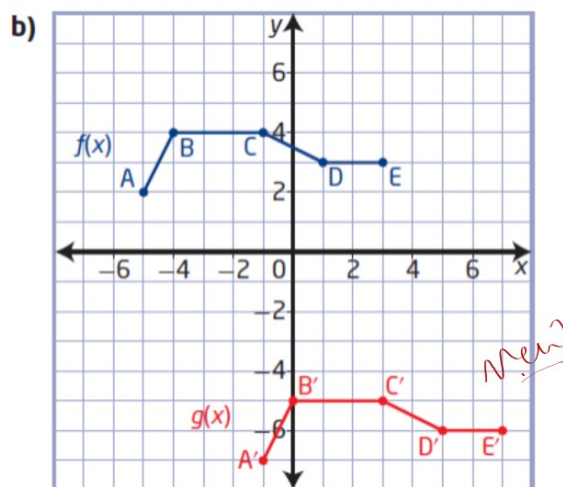
What transforms $f(x)$ to $g(x)$

Solutions

Determine the Equation of a Translated Function



$$y = (x + 4)^2 - 6$$



R4, D9

$$g(x) = f(x - 4) - 9$$

$$y+6 = f(x-3)$$
$$\boxed{y} = f(x-3) - 6$$

R3 D6

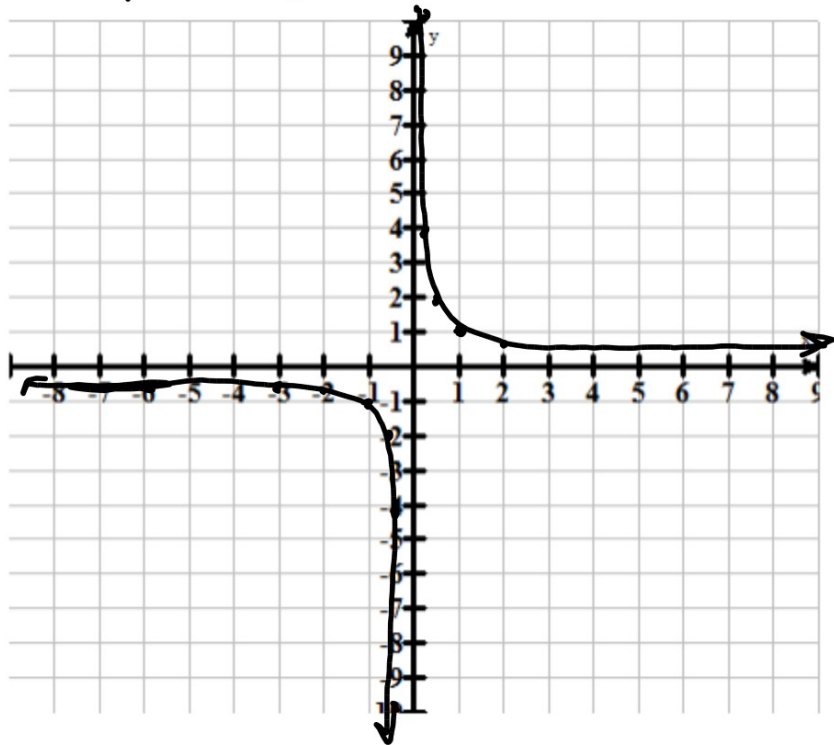
Rearrange
for y

$$y-8 = (x+2)^2$$
$$y = (x+2)^2 + 8$$

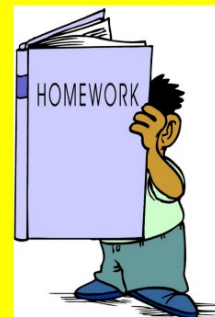
L2, U8

$$y = \frac{1}{x} \quad \text{"Reciprocal Function"}$$

x	y
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
-0.5	-2
-0.25	-4
-0.1	-10
0	undefined
0.1	10
0.25	4
0.5	2
1	1



NOTE: when given $y+8=f(x-6)$
 rewrite as: $y=f(x-6)-8$
 then mapping rule is: $(x,y) \rightarrow (x+6, y-8)$



Today & Tomorrow

Page 12/13 1,3,4, 5, 6 , 7, #8 9a,b 10a,11, 12,
 15, 16

Apply

5. For each transformation, identify the values of h and k . Then, write the equation of the transformed function in the form $y - k = f(x - h)$.
 - a) $f(x) = \frac{1}{x}$, translated 5 units to the left and 4 units up
 - b) $f(x) = x^2$, translated 8 units to the right and 6 units up
 - c) $f(x) = |x|$, translated 10 units to the right and 8 units down
 - d) $y = f(x)$, translated 7 units to the left and 12 units down
6. What vertical translation is applied to $y = x^2$ if the transformed graph passes through the point $(4, 19)$?
7. What horizontal translation is applied to $y = x^2$ if the translation image graph passes through the point $(5, 16)$?

Answers Pg 554-555

$$(a) f(x) = \frac{1}{(x+5)} + 4$$

$$\rightarrow b) f(x) = (x-8)^2 + 6 \checkmark$$

$$y - 6 = f(x - 8)$$

$$(c) f(x) = |x - 10| - 8$$

$$(d) y = f(x + 7) - 12$$

9. The graph of the function $y = x^2$ is translated 4 units to the left and 5 units up to form the transformed function $y = g(x)$.

- Determine the equation of the function $y = g(x)$.
- What are the domain and range of the image function?
- How could you use the description of the translation of the function $y = x^2$ to determine the domain and range of the image function?

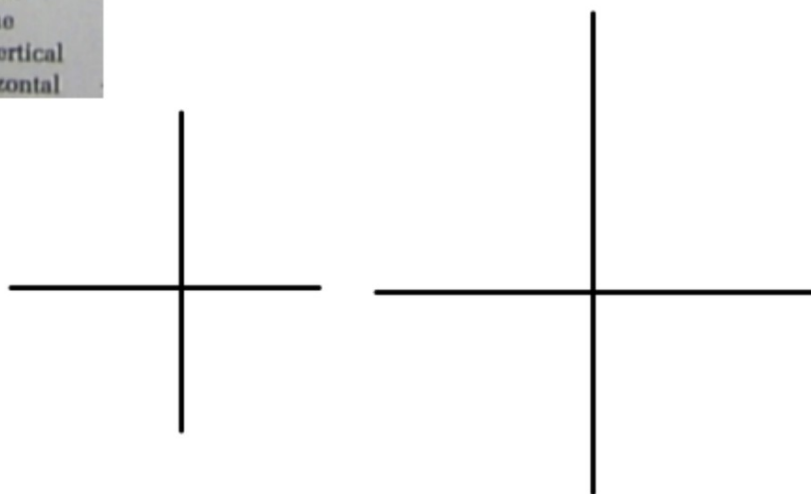
$$y = (x + 4)^2 + 5$$

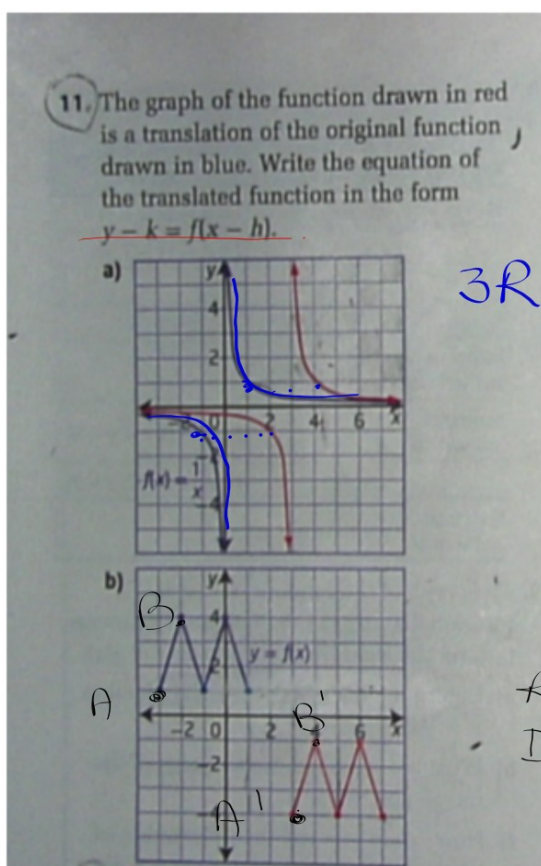
$$D: x \in \mathbb{R}$$

$$R: y \geq 5$$

10. The graph of $f(x) = |x|$ is transformed to the graph of $g(x) = f(x - 9) + 5$.

- Determine the equation of the function $g(x)$.
- Compare the graph of $g(x)$ to the graph of the base function $f(x)$.
- Determine three points on the graph of $f(x)$. Write the coordinates of the image points if you perform the horizontal translation first and then the vertical translation.
- Using the same original points from part c), write the coordinates of the image points if you perform the vertical translation first and then the horizontal





$y = f(x)$ original
R3

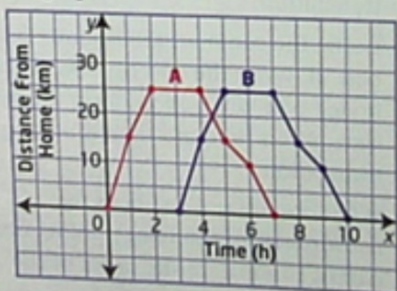
$$y = f(x - 3) *$$

$$y = f(x)$$

$$y = f(x - 6) - 5$$

12. Janine is an avid cyclist. After cycling to a lake and back home, she graphs her distance versus time (graph A).

- If she left her house at 12 noon, briefly describe a possible scenario for Janine's trip.
- Describe the differences it would make to Janine's cycling trip if the graph of the function were translated, as shown in graph B.
- The equation for graph A could be written as $y = f(x)$. Write the equation for graph B.



$$A: y = f(x)$$

$$B: y = f(x-3)$$

16. Paul is an interior house painter. He determines that the function $n = f(A)$ gives the number of gallons, n , of paint needed to cover an area, A , in square metres. Interpret $n = f(A) + 10$ and $n = f(A + 10)$ in this context.

$n = \#$ gallons paint
 $A = \text{area}$

$$n = f(A) + 10$$

$$y = f(x)$$

$$(A, n) \rightarrow (A, n+10)$$

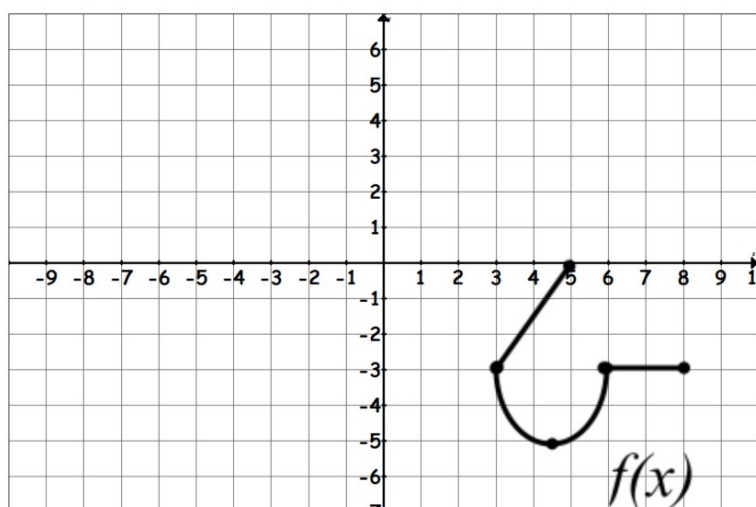
$$(x, y)$$

gallons paint
 increases
 by 10

$$n = f(A+10)$$

$$(A, n) \rightarrow (A-10, n)$$

Area is decreased
 by 10 units



Sketch
 $y=f(x+10)+6$

